Decibels Demystified

This note is inspired by concepts found in *Radiotron Designer's Handbook*, [1] an old reference book edited by F. Langford-Smith and first published in 1934. Besides being a compendium of tube data and old tube circuit design, the book contains fundamental information that is still useful today.

One of those ideas is a simple approach to estimating decibels. Although you no doubt are familiar with decibels, the procedures outlined in this article may be new and useful to you.

Michael Johnson
Raytheon Company

Bel and Decibel Basics

Let's get started with a review of some familiar ideas. Decibels are used for power ratios expressed as a logarithm. In electronics, we use them in logarithmic expressions of a ratio of a given power to a reference power. The defining equation for this logarithmic power ratio is:

$$R_{Bel} = \log_{10}(P/P_{ref}) \tag{1}$$

where R is the power ratio expressed in Bels, P_{ref} = reference power used for calculating the ratio (1 W or 1 mW, etc.), and P is the power level for which the ratio is calculated. For example, a ratio of P = 10 watts and P_{ref} of 1 watt is 1.0 Bel (10^1 ratio). Similarly, the ratio for 100 watts to 1 watt is 2.0 Bels (10^2 ratio).

The basic power ratio, the "Bel," springs from Alexander Graham Bell in his work on hearing. The Bel was devised because the range of human hearing is so wide that expressing values linearly is quite inconvenient, requiring numbers ranging from very small to quite large. By expressing these numbers using logarithms we convert the very large or very small numbers into conveniently sized numbers.

The Bel expression gives rise to values that are largely resolved to the right of the decimal point, so it proved useful to define a *deci*-Bel to move the decimal point, so there are 10 decibels per Bel. (Strictly speaking, we should write "deciBels," but its spelling with all lower case letters is common usage. However, we do observe the correct capitalization when writing the units of notation, dB.)

For decibels, equation (1) is modified to:

$$R_{dB} = 10\log_{10}(P/P_{ref}) \tag{2}$$

The reference power for decibels can be watts, milliwatts, or microwatts, etc., which give rise to dB ratio names that identify the reference (e.g. dBW, dBm, dBµ, etc). Thus, using the above example of the ratio for P = 10 watts and P_{ref} of 1 watt, the ratio is 10 dB, and for P = 100 watts and P_{ref} of 1 watt, the ratio is 20 dB.

Of course, the computation can be reversed, so if we desire a power that is 20 dB greater than our 1 watt reference, we can calculate it to be 100 watts.

The Magic of the Old Ways

Back when the *Radiotron Designer's Handbook* was first published, shortcuts for computing solutions mentally were the order of the day. Without these techniques you had to resort to a slide rule or a book of logarithm tables to determine an arithmetical answer. Today, we rely on calculators in a somewhat thoughtless fashion to convert dBs to power or voltage ratios and vice versa, but we really don't need calculators at all for many of our estimates of decibels.

Langford-Smith presented a technique for doing these conversions mentally that is fairly easy, has the advantage of freeing you from the calculator, and most importantly, provides nice insights into the relationships between the ratios and the equivalent decibels. Let's see what was suggested all those years ago.

The fundamental notion is a simple one. First, let's start with a ratio-to-dB conversion that you already know well, a 2 to 1 power ratio, which we will express as, say, 2 watts relative to 1 watt:

$$R_{dB} = 10\log_{10}(2\text{W}/1\text{W}) = 3.010 \text{ dBW} \tag{3}$$

We can round off the result to 3 dBW, and to create a more general apporach, we can ignore the reference units, and just refer to any 2:1 ratio as "3 dB." Just be sure to keep track of the reference power!

Another well known conversion is a 10:1 ratio, or 10 dB. Starting with these familiar values, let's add the only two other conversions that we need to remember. These are not as familiar, and so need to be memorized:

1 dB → 1.26 : 1 (some round this off to 1.25)
2 dB → 1.58 : 1

We now know four of the power ratios for integer dB values from 1 to 10—which is enough data to easily compute the remaining six values (see Table 1).

Before we go further, we need to be reminded of one other characteristic of decibels versus their ratios, that is, adding decibels is the same as multiplying ratios.

This is a second useful feature of logarithms (besides compressing a large span of numbers into a smaller range), and is used to simplify calculations on both slide rules and when using log tables. Log values are added, the sum is located on the slide rule scale or in the table, then converted back to a numerical value to yield the product of the original two numbers. With the advent of the calculator, these ways to find the products of numbers are now shrouded in the mists of antiquity.

But we resurrect a form of them here. For example, adding 3 dB + 10 dB is the same as multiplying the ratios of 2 and 10. Thus, 13 dB is the same as a ratio of 20/1. That's it! Starting with the few numbers noted above, we can figure out all other integer dB values without resorting to a calculator.

Finding the Rest of the dBs

Let's fill in the rest of Table 1 by finding the sums of dB values and multiplying their ratios.

To find the ratio for 4 dB, recall that 4 dB = 3 dB + 1 dB. Since 3 dB is a ratio of 2.0, and 1 dB is a ratio of 1.26, the ratio for 4 dB is 2.0 × 1.26 = 2.52.

In a similar fashion: 5 dB = 3 dB + 2 dB, so the ratio for 5 dB is 2.0 × 1.58 = 3.16. And so on for 6 dB (3 dB + 3 dB), 7 dB (3 dB + 4 dB), 8 dB (3 dB + 5 dB) and 9 dB (3 dB + 3 dB + 3 dB).

Any dB Value to a Power Ratio

To find the ratio for any dB value above 10 dB, all you need to do is:

1. Split the decibel number into two pieces by subtracting off the units value.
2. Take the units value and determine its ratio, using the technique just described.
3. Then, take the remaining dB value, divide it by ten (simply drop the zero), and use the remaining integer for an exponent of 10, which is the inverse of the log function.
4. The product of the two ratios is the final ratio

For example, take 74 dB and follow the above steps:

1. 74 dB → 70 dB + 4 dB
2. 4 dB → 2.52
3. 70 dB → $10^{(70/10)} = 10^7$
4. 74 dB → 2.52×10^7

Now, if the decibel value is negative, then we take the product of the inverse of the ratios:

1. –33 dB → –3 dB + (–30 dB)
2. –3 dB → 1/2.0
3. –30 dB → $1/10^3 = 10^{-3}$
4. –33 dB → $1/2 \times 1/10^3 = 0.5 \times 10^{-3}$ or 5×10^{-4}

Ready for Voltage Ratios?

Now, if you work with voltages, the ratios are obtained with a similar technique, but we need to remember that power is a function of the voltage squared. The exponent of 2 means that the logarithmic ratios are all doubled, compared to power. Thus, the basic decibel equation for a voltage ratio is:

$$R_{dB} = 20\log_{10}(V/V_{ref}) \qquad (4)$$

The ratio for a 2:1 voltage ratio is 6 dB, and for a 10:1 voltage ratio is 20 dB. A table of integer values can be made for voltage-based dB as well. The ratios for Table 1 can be used as a starting point, but with dBs double those

dB	Ratios (initial)	Ratios (final)
1	1.26	1.26
2	1.58	1.58
3	2.0	2.0
4	—	2.52
5	—	3.16
6	—	4.0
7	—	5.0
8	—	6.32
9	—	8.0
10	10.0	10.0

Table 1 · dB values and associated power ratios. The left column includes the "easy" 3 dB and 10 dB values, plus the only other two that must be memorized to obtain all ten integer dB values.

of the power ratios (1, 2, 3, ... , 10 dB values become 2, 4, 6, ... , 20 dB). The intermediate values can be calculated as before, using the additional memorized values of

1 dB → 1.12
3 dB → 1.41
5 dB → 1.8

The process of calculating larger dB values must be referenced to 20 dB, that is, the portion of the number split off must be divisible by 20. Using the 74 dB example from before:

1. 74 dB → 60 dB + 14 dB [60 is divisible by 20]
2. 14 dB → 5.0 [14 dB = $20\log_{10}(5.0)$]
3. 60 dB → $10^{(60/20)} = 10^3$ [divide 60 by 20]
4. 74 dB → 5.0×10^3 [multiply the ratios]

Well, there it is. With a little practice, you can go from dBs to ratios (and vice versa) fairly easily, all thanks to those who went before us, like Langford-Smith, who pointed out the way.

Reference

1. F. Langford-Smith, editor, *Radiotron Designer's Handbook*, Fourth Edition, 1952, Wireless Press for Amalgamated Wireless Valve Company, Sydney, Australia, Reproduced and Distributed by the Radio Corporation of America. [*Also available as a reprint, titled* Radio Designer's Handbook, *from Newnes, an imprint of Butterworth-Heinemann, 1997, ISBN 0-7506-3635-1 —ed.*]

Author Information

This note was submitted by Michael E. Johnson of Raytheon Company. He can be reached by e-mail at Michael_E_ Johnson@raytheon.com.